Motion of Spherical Particles in a Bingham Plastic

RALPH W. ANSLEY
and
TERENCE N. SMITH
University of Adelaide, Adelaide, Australia

This paper describes an investigation of the settling characteristics of spherical particles in Bingham plastic fluids. A stress analysis is used to derive a nondimensional parameter which describes the laminar settling regime. The terminal velocities of spheres in the Reynolds number range of 2 to 130 have been determined. These, together with published data at higher Reynolds numbers, are shown to substantiate the theoretical argument.

A correlation of drag coefficient as a function of the settling parameter indicates the transition from laminar to turbulent settling.

There is current interest in the utilization of homogeneous suspensions of fine sized solids in the fluid transport of heterogeneous mixtures of coarse solid particles. It has been established that the introduction of the fines beneficially affects the transport of coarse solids (*1, 2*). Although the homogeneous suspensions have been identified as non-Newtonians, there have not as yet been satisfactory solids pipelining design criteria for the transport of coarse particles based on the rheological properties of fines suspensions. A recent investigation (*3*) has shown that the use of existing design formulas, all of which include the coarse solids settling characteristics as parameters, can result in significantly erroneous design unless these parameters are determined in the fluid actually used to transport these solids. This conclusion, based on limited experimental evidence, prompted a more detailed study of settling in non-Newtonian suspensions, particularly in the Stokes and intermediate settling regimes.

EXPERIMENTAL PROCEDURE

Considerable preliminary work was necessary to find a fluid with stable Bingham properties, suitable for the settling tests. Although there have been numerous investigations of homogeneous suspensions in both pipelines and viscometers in which it has been suggested that the rheological behavior of these suspensions can be approximated by the idealized Bingham model (*4* to *6*), most suspensions develop a density gradient after standing in a column and exhibit a nonlinear relation between shear stress and rate of shear at rates of shear approaching zero. Furthermore, to extend the range of variables, it was desirable to choose a plastic material with properties distinctly different from those used by others (*3, 7*). Tomato sauce (catsup), which had been suggested (*8*) as a Bingham plastic, was investigated; it was found suitable and used in the tests. The rheological properties of a Bingham plastic fluid are defined by

$$\tau = \tau_y + \eta \frac{du}{dr} \tag{1}$$

The Bingham plastic properties of the tomato sauce were substantiated by a series of flow tests in tubes. Testing with a rotary viscometer was discontinued when the sauce showed a tendency to bleed after continuous shear of the material in contact with the viscometer. The tube viscometer offered much greater similarity to the subsequent settling tests in which the spheres continuously encountered material which had not recently undergone shear. Flow rates of the material under gravitational acceleration and under applied pressures were measured in stainless steel tubes ranging in internal diameter from 0.366 to 1.709 cm. to provide a similar scale to the subsequent drop tests. The results of these tests were analyzed with the aid of the Buckingham equation in the form

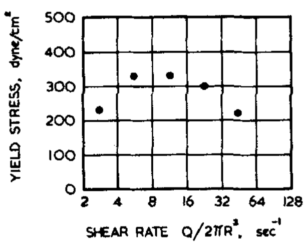
$$\frac{8Q}{\pi R^4 \dfrac{dP}{dL}} = \frac{1}{\eta}\left[1 - \frac{4}{3}\left[\frac{2\tau_y}{R\dfrac{dP}{dL}}\right] + \frac{1}{3}\left[\frac{2\tau_y}{R\dfrac{dP}{dL}}\right]^4\right] \tag{2}$$

to yield values of τ_y and η from the coefficients in the least-squares fit to the regression equation. The 102 data points were grouped variously in intervals of shear rate and tube transit time to test for deviations from ideal plastic fluid behavior. The variation of yield stress values was small over the ranges of shear rate and time investigated, and is shown in Figures 1*a* and 1*b*. Repeated tube and drop tests indicated that the sauce did not deteriorate during the experimental program.

The values of τ_y and η selected from the tube tests for use in analysis of the settling data were 290 dynes/sq. cm. and 0.67 poise. The density of the tomato sauce was 1.156 g./cc.

For the settling tests, silver balls were cast in various sizes and then neutron-irradiated to give a gamma activity of about 1 mC. of Ag 110 per sphere. The tests were made in a 6 in. × 6 in. × 54 in. high acrylic column. Spheres were dropped by hand at the surface of the fluid in the column and were recovered from the conical bottom by permitting them to flow out with some fluid via a 3/4-in. discharge tube. Two collimated Geiger-Müller tubes set 18 in. vertically apart in the lower part of the column and connected to a radiation counter and continuous recorder permitted measurement of the time interval required for the settling sphere to traverse this distance. Effective mixing of the fluid in the column was achieved by a circulating pump withdrawing fluid from two different points in the column and introducing it at the bottom.

The results of the settling tests are summarized in Table 1. These values are the best of, in most cases, about twenty drop tests.



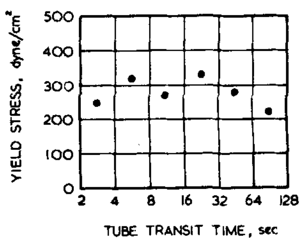


Fig. 1*a* (*left*) and 1*b* (*right*). Flow tests with plastic fluid. Variation of yield stress with shear rate and with tube transit time.

TABLE 1. SUMMARY OF BALL DROP TESTS

Diameter, cm.	Weight, g.	Density, g./cc.	Settling velocity, cm./sec.
0.617	1.196	9.73	3.23
0.681	1.721	10.41	7.09
0.775	2.451	10.06	8.53
0.932	3.861	9.35	13.0
1.095	6.873	9.98	40.1
1.265	10.649	10.04	56.5
0.950	2.631	5.84	2.54
1.082	4.097	6.15	5.37
1.270	7.215	6.72	17.9
1.244	5.070	5.02	4.47

The repeatability of the settling velocities in the tests varied with velocity itself. The standard deviation ranged from 3% for the fastest balls to 10% for the slowest. Considerable effort was devoted to the elimination of all probable causes of the spread of results in the case of the slowly settling spheres, such as the presence of air bubbles, the segregation of sauce material, and the continued working of a column of material. No reduction in variability could be made.

The data listed in Table 1 cover a range of 2 to 130 of modified Reynolds number (in terms of the coefficient of rigidity). Results for spheres settling in clay suspensions for a modified Reynolds number range of 70 to 1,250 are available (*3*). The original program to obtain additional data over the full range of laminar settling was curtailed upon the recent publication of the results of Valentik and Whitmore (*7*). Their extensive data for spheres settling in kaolin suspensions cover a range of 7 to 11,500 modified Reynolds number.

Both duPlessis and Ansley (*3*) and Valentik and Whitmore (*7*) have used a yield stress obtained by extrapolating the linear portion of shear stress-rate of shear curve to an ordinate intercept. However, all suspensions used in their settling tests have marked nonlinearity at low shear rates. Since the settling tests involved low rates of shear (defined as u/D), the slope of a tangent to the curve at a characteristic shear rate for the particular settling tests has been used as the coefficient of rigidity and its ordinate intercept as the yield stress.

FLOW MODEL

Some consideration was given to the possibility of an analytic solution of the flow of a sphere in a Bingham plastic but no progress could be made. Conventional dimensional analysis or stress analysis leads to the general relation between the variables in the particle-plastic fluid flow system

$$C_D = \phi\left(\frac{Du\rho}{\eta}, \frac{D\tau_y}{u\eta}\right) \qquad (3)$$

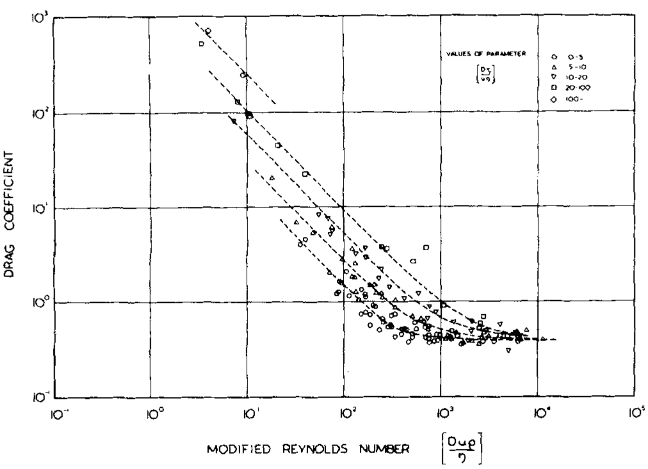


Fig. 2. Drag coefficents of spheres in plastic fluids as function of modified Reynolds number.

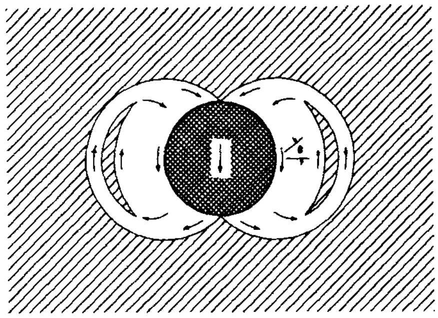


Fig. 3. Postulated flow pattern of plastic fluid about solid sphere.

which is identical to the correlation of friction factor for flow of plastic fluid in pipes obtained by Hedstrom (9).

Figure 2 shows a plot of the settling data from the three sources in accordance with Equation (3). The drag coefficient C_D is plotted as ordinate against the modified Reynolds number $Du\rho/\eta$ as the abscissa and the remaining term $D\tau_y/u\eta$ shown as a parametric value.

A useful simplification of the drag coefficient correlation can be obtained if the two terms on the right side of Equation (3) can be combined into a single parameter. Consideration of the mechanics of the sphere-plastic fluid flow system suggests a basis for the formulation of such a parameter.

The flow system postulated in this analysis is depicted in Figure 3. The stress distribution imposed on the plastic material by the motive force on the sphere causes the material to become fluid in an envelope surrounding the sphere. Within the envelope, the motions of the sphere and the displaced fluid are steady as the sphere-envelope system moves through the plastic material causing instantaneous, localized transformation between the plastic and fluid states.

The shape of the fluid envelope is derived from suppositions regarding the slip-line fields set up by stresses imposed on cavities in isotropic plastic materials. The yield envelope proposed here is a kind of truncated toroid with its section centered on the surface of the sphere and having a diameter $\sqrt{2}D$. A general discussion of slip-line field theory is given by Hill (*10*).

The motive force on the sphere is opposed by the resultants of the normal pressure field and the shear stress field associated with the maintenance of the fluid envelope and by the drag resulting from the relative motion of the sphere and the fluid in the envelope.

The possibility of describing the dynamic state of the system by a single parameter arises in the following manner. The fluid surrounding the sphere moves under the influences of a characteristic inertial stress ρu^2, a "viscous" stress $\eta u/D$, and a yield stress τ_y. If the last two are regarded as "restraining" stresses, the dynamic state of the fluid can be expressed in the ratio $(\rho u^2)/(\eta u/D + K\tau_y)$, where K is a proportionality factor. To apply this ratio as a correlation parameter, it is necessary to fix the value of K. An obvious selection is the ratio of the contributions of the yield and viscous stress terms to the drag sphere.

The drag contributions of the yield stress and viscous stress terms can be evaluated from a consideration of the proposed flow model.

The yield envelope about the moving sphere is maintained in the fluid state by a pressure distribution which imposes, in a series of surfaces, shear stresses equal to the plastic yield stress. The plastic material is then free to slip, or move, parallel to these surfaces. In the proposed model, the slip surfaces are coaxial about the equator of the sphere. Within every slip surface, certain necessary relations between the normal stresses and the yield stress can be formulated. For two-dimensional slip in a plastic material, the normal pressure p varies along a slip-line with the angle θ, of the normal to the slip-line such that $\delta p = 2\ \tau_y\ \delta\theta$. For three-dimensional axisymmetrical slip, the pressure variation within a slip surface is described by this term and additional terms due to circumferential expansion (see reference *10*). In the proposed model, however, there is not only symmetry about the axis of the sphere, but also symmetry about the equator. Accordingly, the circumferential terms on either side of the equator should be equal in magnitude but opposite in direction. In the integral of the pressure variation over the surface of the solid sphere, therefore, these terms should vanish so that only the integral of the two-dimensional variation with the angle θ remains. This integral is the drag contributed by the pressure field maintaining the envelope in a fluid state and has the value $5\pi^2 D^2 \tau_y/8$.

Superimposed on the yield envelope pressure system is the stress field associated with the relative motion of the solid sphere and the fluid material within the envelope. With the dynamic effects induced close to the sphere approximated as those due to a Stokes flow field, the viscous drag on the sphere is $3\pi D u \eta$. The magnitudes of the shear stresses at the surface of the sphere, however, must exceed the Stokes flow shear stresses, which are derived from velocity gradients, by the value of the yield stress. The resultant, in the axial direction, of the yield stress acting over the surface of the sphere is $\pi^2 D^2 \tau_y/4$.

The value of the drag term involving yield stress is then $7\pi^2 D^2 \tau_y/8$ and that of the term involving viscous stress is $3\pi D u \eta$. The value of K in the dynamic parameter is the ratio of the respective coefficients of τ_y and η in these terms and is $7\pi/24$.

Figure 4 shows all data plotted in terms of drag coefficient and the dynamic parameter $(\rho u^2)/(\eta u/D + 7\pi/24\ \tau_y)$.

DISCUSSION

The theoretical argument shows excellent agreement with experimental results. A strong correlation between the drag coefficient and the dynamic parameter is readily evident in Figure 4. The 141 pieces of data shown in Figure 4 include results for spherical particles ranging in size from 0.25 cm. in diameter to 5.71 cm. with densities over the range of 1.3 to 10.4 g./cc. The ten fluids used in the tests had ranges of 5 to 300 dynes/sq. cm. yield stress and 0.05 to 2.78 poises coefficient of rigidity. In general, high yield stresses tended to be associated with high coefficients of rigidity.

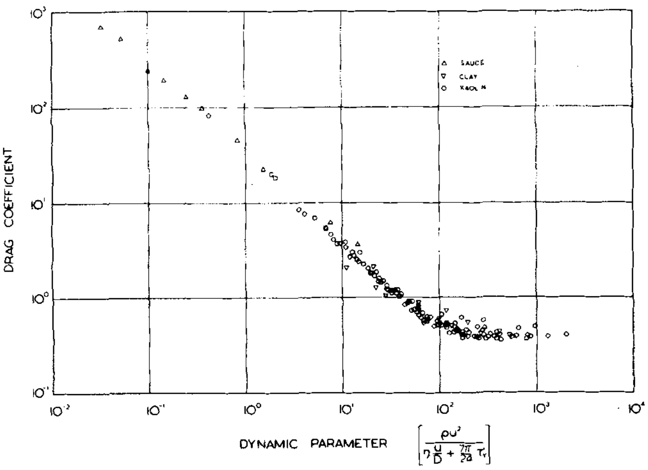


Fig. 4. Drag coefficients of spheres in plastic fluids as function of proposed dynamic parameter

TABLE 2. COMPARISON OF WEIGHTS AND SUPPORT DUE TO YIELD STRESS

Diameter, cm.	Weight, g.	$5\pi^2 D^2 \tau_y/8$, g.	$7\pi^2 D^2 \tau_y/8$, g.	Settling velocity, cm./sec.
0.487	0.648	0.45	0.63	0.30
0.617	1.196	0.72	1.01	3.2
0.937	2.003	1.66	2.32	nil
1.094	2.903	2.26	3.16	0.18

The laminar settling characteristics of spheres in a non-Newtonian fluid exhibiting Bingham model rheological properties are uniquely defined by the correlation between drag coefficient and the dynamic parameter as shown in Figure 4. The dynamic parameter is a nondimensional measure of the stress condition of the particle-plastic fluid system, similar to the conventional Reynolds number which is correlated with drag coefficient to describe settling of spheres in Newtonian fluids. In fact, Figure 4 reduces to the C_D-N_{Re} correlation if the fluid has a simple viscosity with zero yield stress. The transition from laminar to turbulent settling in a Bingham plastic is well defined by the correlation and corresponds to the same value of drag coefficient as for the Newtonian case.

The physical flow model, while useful in deriving the dynamic parameter, cannot be supported by direct evidence. Indeed, there can be no assurance that the fluid envelope in motion remains the same in shape or size as the static yield envelope shown in Figure 3. Furthermore, the assumptions about the pressure distribution in the envelope and the Stokesian flow field are, at best, plausible approximations. However, it certainly provides a close quantitative estimate of the effect of yield stress in the sphere-plastic fluid flow system. Some additional substantiation of the magnitude of the proposed yield stress effect is derived from a consideration of certain spheres which either just settled or just failed to settle in the experimental program. On a stationary sphere, the support force due to the yield stress of the plastic material exceeds the weight of the sphere. Here, the weight is the dead weight and not the buoyant weight in fluid material. The maximum support force that the plastic material can offer is somewhere between $5\pi^2 D^2 \tau_y/8$, the support available from normal pressure alone and $7\pi^2 D^2 \tau_y/8$, which includes the integral of the yield stress over the sphere surface. The distribution of shear stress over the surface of the stationary sphere is not known but must, of course, be everywhere less than the yield stress in value. Table 2 compares the weights of four spheres which just moved or just did not move with these limits of the maximum support force.

CONCLUSIONS

Current developments in fluid transport of solids require a method to determine the settling characteristics of solids in fluids which exhibit Bingham plastic rheological characteristics. A stress analysis has been used to obtain a nondimensional expression which is a measure of the stress condition for the system of the motion of a spherical particle in a Bingham plastic. A flow model has been used to determine the relative effects of yield stresses and viscous stresses. The theory is shown to be in excellent agreement with extensive experimental data.

NOTATION

C_D = drag coefficient $\left(= \frac{4}{3} \frac{(\rho_s - \rho) gD}{\rho u^2}\right)$
D = diameter of sphere
du/dr = velocity gradient in fluid
g = gravitational acceleration
K = constant of proportionality
L = length of viscometer tube
N_{Re} = Reynolds number
P = pressure
p = an arbitrary normal pressure
Q = volume flow rate
R = radius of viscometer tube
u = velocity of solid particle through fluid

Greek Letters

η = coefficient of rigidity
ϕ = function
ρ = density of fluid
ρ_s = density of solid
τ = fluid shear stress
τ_y = plastic yield stress
θ = angle of normal to slip-surface in flow model

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Prediction of Pressure Drop for Two-Phase, Two-Component Concurrent Flow in Packed Beds

JIM L. TURPIN and R. L. HUNTINGTON
University of Oklahoma, Norman, Oklahoma

Two-phase, gas-liquid concurrent flow in packed beds was investigated with the use of an air-water system and 2-, 4-, and 6-in. diameter columns packed with tabular alumina particles of 0.025 and 0.027 ft. diameters. Total pressure drop, column operating pressure, and liquid saturation were measured as functions of gas flow rate, fluid temperatures, and flow direction at several constant liquid flow rates for each column.

Correlation of the frictional pressure loss was achieved in terms of a defined two-phase friction factor and a second correlating parameter which is a function of the liquid and gas Reynolds numbers. A viscosity correction factor was required to extend the friction factor correlation to include liquid viscosities widely divergent from that of water.

The liquid saturation data for both upward and downward flow were correlated in terms of the ratio of mass flow rates of the respective phases.

The general field of multiphase flow has received much attention in recent years because of its widespread occurrence in engineering operations. It is encountered in such basic areas as distillation, evaporation, heat transfer, gas absorption, and other areas of the chemical processing industry. The most common type of multiphase flow involves gas and liquid phases, and the term two-phase flow will be taken as the gas-liquid combination in this paper.

Much effort has been expended for research on two-phase flow, although only a very minute portion of this research has been concerned with two-phase concurrent flow in packed beds, the reason for which was the apparent lack of a practical application for this work. However it has been shown (*1*) that under certain conditions, concurrent gas absorption is a more desirable operation than is gas absorption utilizing countercurrent flows. With countercurrent operation, flow rates are limited by the flooding point of the column, while the only limit to flow rates in concurrent operation is the amount of power to be expended in forcing the fluids through the column, thus providing a

Jim L. Turpin is at the University of Arkansas, Fayetteville, Arkansas.